Homework Assignment #7

Note

This assignment is due 2:10PM Monday, November 26, 2018. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in the Algorithms box placed in the IM office on the 7-th floor of Management College Building 1. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are four problems in this assignment, each accounting for 25 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (7.23) Describe an efficient implementation of the Eulerian graph algorithm discussed in Section 7.2. The algorithm should run in linear time and space. (Given an undirected connected graph G=(V,E) such that all the vertices have even degrees, find a closed path P such that each edge of E appears in P exactly once.)

2. (7.27) Design an algorithm to find a vertex in a connected undirected graph whose removal does not disconnect the graph. The algorithm should run in linear time. (Do not use the biconnected components algorithm.) As a consequence, prove that every connected graph contains such a vertex.

3. (7.25) A *directed Eulerian circuit* is a directed circuit that contains each edge exactly once. Prove that a directed graph contains a directed Eulerian circuit if and only if the indegree of each vertex is equal to its outdegree, and the underlying undirected graph is connected. Design an efficient algorithm to find such an Eulerian circuit if it exists

4. (7.52) Find necessary and sufficient conditions under which the set of edges E of an undirected graph $G = (V, E)$ can be partitioned into disjoint subsets $E_1, \ldots, E_k$ such that each E_i corresponds to a simple cycle. Design an efficient algorithm to find such a partition in graphs that satisfy these conditions.