Homework Assignment #5

Note

This assignment is due 2:10PM Monday, October 29, 2018. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in the Algorithms box placed in the IM office on the 7-th floor of Management College Building 1. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. Apply the `Quicksort` algorithm to the following array.

9	2	12	1	13	10	5	3	4	6	7	14	11	15	8

   Show the result after each partition operation similar to Figure 6.12 in [Manber 1989].

2. Build a heap using the bottom up approach from the array given in Problem 1. Show the result after each step similar to Figure 6.15 in [Manber 1989].

3. (6.29) The input is d sequences of elements such that each sequence is already sorted, and there is a total of n elements. Design an $O(n \log d)$ algorithm to merge all the sequences into one sorted sequence. Justify the $O(n \log d)$ complexity of your algorithm.

4. (6.34) The input is a heap of size n (in which the largest element is on top), given as an array, and a real number x. Design an algorithm to determine whether the kth largest element in the heap is less than or equal to x. The worst-case running time of your algorithm should be $O(k)$, independent of the size of the heap. You can use $O(k)$ space. (Notice that you do not have to find the kth largest element; you need only determine its relationship to x.)

5. (6.41) Use the idea of the information-theoretic bound to prove a lower bound of $\Omega(\log n)$ comparisons for the problem of finding the value of i such that $x_i = i$ in the sequence $x_1, x_2, \ldots, x_n$, or determining that no such i exists.