Homework Assignment #1

Note

This assignment is due 2:10PM Tuesday, September 25, 2018. Please write or type your answers on A4 (or similar size) paper. Drop your homework by the due time in the Algorithms box placed in the IM office on the 7-th floor of Management College Building 1. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

There are five problems in this assignment, each accounting for 20 points. You must use *induction* for all proofs. (Note: problems marked with "(X.XX)" are taken from [Manber 1989] with probable adaptation.)

1. (2.3) Find the following sum and prove your claim (i.e., guess and verify by induction):

$$1 \times 2 + 2 \times 3 + \cdots + n(n+1).$$

2. (2.10) Find an expression for the sum of the i-th row of the following triangle, which is called the **Pascal triangle**, and prove *by induction* the correctness of your claim. The sides of the triangle are 1s, and each other entry is the sum of the two entries immediately above it.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

3. The Harmonic series $H(k)$ is defined by $H(k) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k-1} + \frac{1}{k}$. Prove that $H(2^n) \geq 1 + \frac{n}{2}$, for all $n \geq 0$ (which implies that $H(k)$ diverges).

4. Consider binary trees where every internal node has two children. For any such tree T, let l_T denote the number of its leaves and m_T the number of its internal nodes. Prove *by induction* that $l_T = m_T + 1$.

5. (2.36) Let $a_1, a_2, \cdots, a_n$ be positive real numbers such that $a_1 a_2 \cdots a_n = 1$. Prove *by induction* that $(1 + a_1)(1 + a_2) \cdots (1 + a_n) \geq 2^n$. (Hint: In the inductive step, try introducing a new variable that replaces two chosen numbers from the sequence.)