演算法 #7

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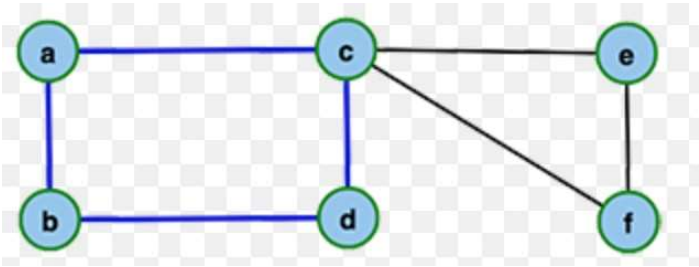
Q1.

- Describe an efficient implementation of the Eulerian graph algorithm discussed in Section 7.2. The algorithm should run in linear time and space. (Given an undirected connected graph G=(V,E) such that all the vertices have even degrees, find a closed path P such that each edge of E appears in P exactly once.)

Q1.




```
Algorithm Eulerian_Path(G, v){
    Input: G = (V, E)
    initial Path
    if any degree of edge is odd or 0:
        return false;
    else:
        Euler(G, v)
    return Path
}
```

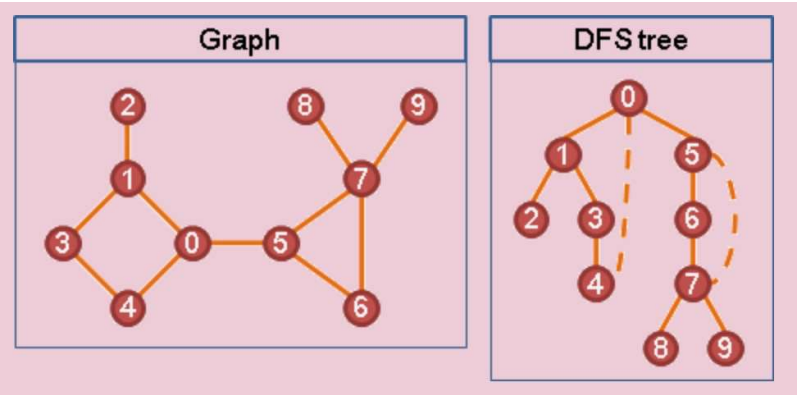
```
Algorithm Euler(G, v):
    for (v, w) in G:
        remove (v, w) in G
        Euler(G, w)
    add v to Path
```

Q2.

- Design an algorithm to find a vertex in a connected undirected graph whose removal does not disconnect the graph. The algorithm should run in linear time. (Do not use the biconnected components algorithm.) As a consequence, prove that every connected graph contains such a vertex.

Q2.

- Apply DFS on connected undirected graph, all the leaves are answer candidates. Because we know that if we remove the DFS tree's leaf, the remaining tree still be connected. And DFS tree must has more than one leaves.
- We can apply DFS on any connected undirected graph, so we know every connected graph contains such a vertex.



Q3.

- A *directed Eulerian circuit* is a directed circuit that contains each edge exactly once. Prove that a directed graph contains a directed Eulerian circuit if and only if the indegree of each vertex is equal to its outdegree, and the underlying undirected graph is connected. Design an efficient algorithm to find such an Eulerian circuit if it exists

Q3.

- Prove a directed Eulerian Circuit -> indegree = outdegree & conntected
  - 當traverse這個graph時，進出一個點的次數會相同並且每個邊只被走一次
    - Indegree = outdegree
  - 並且因爲要能走到每一個點
    - Connected

Q3.

- Prove indegree = outdegree & conntected -> a directed Eulerian Circuit
  - Assume G = (V, E) contains m vertices
  - Inductive Hypothesis: 一個 < m 個點的有向圖，並且其 connected 且 indegree = outdegree，其包含 Eulerian Circuit 且我們知道如何找到這個 circuit
  - 隨機選一個點 v 開始 traverse，直到找到一個 circle P 回到自己（因 indegree = outdegree, connected），並將此 P 從 G 移除，則得到 G' 其 vertices 數量小於 m 且仍然indegree = outdegree, G'_1, G'_2… 分別為 connected graph
  - 根據 Inductive Hypothesis 我們知道如何找到 G'_1, G'_2… 的 Eulerian circuit
  - 重新 traverse circle P，當經過某個 G'_i 就進去 traverse 其 Eulerian circuit，當走回 v 時這個路徑就是 G 的 Eulerian circuit，得證

Q4.

- Find necessary and sufficient conditions under which the set of edges E of an undirected graph *G = (V, E)* can be partitioned into disjoint subsets $E_1, \ldots, E_k$ such that each E_i corresponds to a simple cycle. Design an efficient algorithm to find such a partition in graphs that satisfy these conditions.

Q4.

- 條件：Eulerian Graph
- 因為Eulerian Graph至少包含一個circle，從中刪除一個circle，便依然會剩下一個Eulerian Graph
- Algorithm：
  - 利用Q1的方法找到一個close path P
  - 從P中截出環的部分，剩下的也依然會是環
  - Ex: (A,B) (B,C) (C,D) (D,E) (E,C) (C,F) (F, A)