Homework Assignment #9

Note

This assignment is due 2:10PM Monday, May 20, 2019. Please write or type your answers on A4 (or similar size) paper. Put it on the instructor's desk before the class on the due date starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Exercise 4.9; 10 points) Review the way that we define sets to be of the same size in Definition 4.12 (i.e. Definition 10 in the slides). Show that "are of the same size" is an equivalence relation.

2. (Problem 4.12; 10 points) Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

3. (Problem 4.18; 20 points) A *useless state* in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

4. (Problem 4.31; 20 points) Let $\mathit{INFINITE}_{\mathrm{PDA}} = \{\langle M\rangle \mid M \text{ is a PDA and } L(M) \text{ is infinite}\}$. Show that $\mathit{INFINITE}_{\mathrm{PDA}}$ is decidable.

5. (Exercise 5.1; 20 points) Show that EQ_{CFG} is undecidable.

6. (Exercise 5.4; 20 points) If A is reducible to B and B is a regular language, does that imply that A is a regular language? Why or why not?