Homework Assignment #5

Note

This assignment is due 2:20PM Monday, April 1, 2019. Please write or type your answers on A4 (or similar size) paper. Put it on the instructor's desk on the due date before the class starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2006, 2013] with probable adaptation.)

1. (Exercise 2.1; 20 points) Consider the following CFG discussed in class, where for convenience the variables have been renamed with single letters.

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T \times F \mid F \\
F &\rightarrow (E) \mid a
\end{aligned}
$$

   Give (leftmost) derivations and parse trees for the following strings.

   (a) $a \times (a + a)$

   (b) $(a + (a))$

2. (Exercise 2.4; 20 points) Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.

   (a) $\{w \mid \text{the length of } w \text{ is a multiple of 3}\}$

   (b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

3. (Exercise 2.6b; 10 points) Give a context-free grammar that generates the complement of the language $\{a^n b^n \mid n \geq 0\}$.

4. (Exercise 2.8; 10 points) Show that the string "`the boy likes the girl with a flower`"

has two different leftmost derivations in the following CFG.

$$
\begin{aligned}
\langle\text{SENTENCE}\rangle &\rightarrow \langle\text{NOUN-PHRASE}\rangle\langle\text{VERB-PHRASE}\rangle \\
\langle\text{NOUN-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-NOUN}\rangle \mid \\
& \quad\ \langle\text{CMPLX-NOUN}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{VERB-PHRASE}\rangle &\rightarrow \langle\text{CMPLX-VERB}\rangle \mid \\
& \quad\ \langle\text{CMPLX-VERB}\rangle\langle\text{PREP-PHRASE}\rangle \\
\langle\text{PREP-PHRASE}\rangle &\rightarrow \langle\text{PREP}\rangle\langle\text{CMPLX-NOUN}\rangle \\
\langle\text{CMPLX-NOUN}\rangle &\rightarrow \langle\text{ARTICLE}\rangle\langle\text{NOUN}\rangle \\
\langle\text{CMPLX-VERB}\rangle &\rightarrow \langle\text{VERB}\rangle \mid \langle\text{VERB}\rangle\langle\text{NOUN-PHRASE}\rangle \\
\langle\text{ARTICLE}\rangle &\rightarrow \texttt{a} \mid \texttt{the} \\
\langle\text{NOUN}\rangle &\rightarrow \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle\text{VERB}\rangle &\rightarrow \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle\text{PREP}\rangle &\rightarrow \texttt{with}
\end{aligned}
$$

5. (Exercise 2.9; 20 points) Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

6. (Exercise 2.14; 20 points) Convert the following CFG (where A is the start variable) into an equivalent CFG in Chomsky normal form.

$$
\begin{aligned}
A &\rightarrow BAB \mid B \mid \varepsilon \\
B &\rightarrow 10 \mid \varepsilon
\end{aligned}
$$