

Homework Assignment #4

Note

This assignment is due 2:20PM Monday, March 25, 2019. Please write or type your answers on A4 (or similar size) paper. Put it on the instructor's desk on the due date before the class starts. Late submission will be penalized by 20% for each working day overdue. You may discuss the problems with others, but copying answers is strictly forbidden.

Problems

(Note: problems marked with "Exercise X.XX" or "Problem X.XX" are taken from [Sipser 2013] with probable adaptation.)

1. (Problem 1.31; 10 points) For languages A and B , let the *perfect shuffle* of A and B be the language $\{w \mid w = a_1b_1 \cdots a_kb_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}$. Show that the class of regular languages is closed under perfect shuffle.
2. (Problem 1.32; 20 points) For languages A and B , let the *shuffle* of A and B be the language $\{w \mid w = a_1b_1 \cdots a_kb_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}$. Show that the class of regular languages is closed under shuffle.
3. (Problem 1.38; 20 points) Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Here, Σ_2 contains all columns of 0s and 1s of length two. A string of symbols in Σ_2 gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$C = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is three times the top row}\}.$$

For example, $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C$, but $\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C$. Show that C is regular. (You may assume the result claimed in Problem 5 of HW#2.)

4. (Problem 1.40; 10 points) Let Σ_2 be the same as in Problem 3. Consider the top and bottom rows to be strings of 0s and 1s and let

$$E = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}.$$

Show that E is not regular.

5. (Problem 1.42; 20 points) Let $C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$. Show that for each $n \geq 1$, the language C_n is regular.

6. (Problem 1.43; 10 points) An *all*-NFA M is a 5-tuple $(Q, \Sigma, \delta, q, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be after reading input x is a state from F . Note, in contrast, that an ordinary NFA accepts a string if *some* state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.
7. (Problem 1.51; 10 points) Prove that the language $\{w \in \{0,1\}^* \mid w \text{ is not a palindrome}\}$ is not regular. You may use the pumping lemma and the closedness of the class of regular languages under union, intersection, and complement. (Note: a *palindrome* is a string that reads the same forward and backward.)